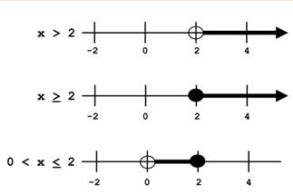
Algebra - Higher

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

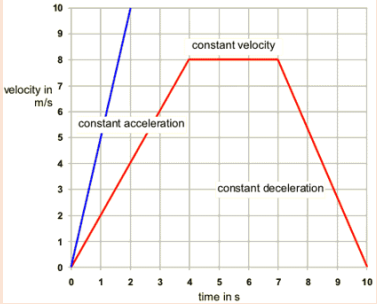
Linear Inequalities



Open circle: < / >

Closed circle: ≤ / ≥

Velocity / Time Graphs



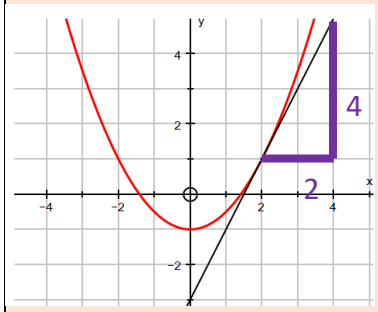
Gradient = acceleration

Area = distance travelled

Iteration – showing a root lies between 2 points:

If there is **a change in sign** for *y* for two particular values of *x* then we can say there is **a root** between these values of *x* and we can say that the equation f(*x*) = 0 will have a solution between these two values of *x*.

Gradients of curves



Gradient of a curve at a point = gradient of the tangent at the point

Algebriac proof – toolkit

Even numbers: 2n, 2n+2, 2n+4,…
Odd numbers: 2n+1, 2n+3, 2n+5,…
Sum: add
Product: multiply
Difference: subtract
Show it's a multiple: factorise
Show it's even: show it's a multiple of 2
Show it's odd: show it's a multiple of 2, plus 1

Completing the square

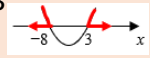
Quadratic expression factorised by completing the square:

$$(x + a)^2 + b$$

Turning point of graph occurs at $(-a, b)$

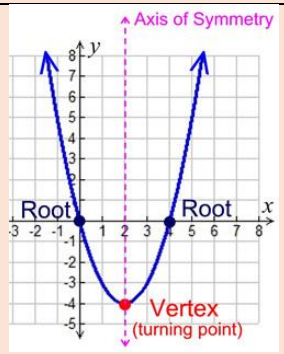
Solve quadratic inequalities

e.g solve $x^2 + 5x - 24 \geq 0$

1. Factorise: $(x + 8)(x - 3) \geq \mathbf{0}$
2. Solve: $x = -8$, $x = 3$
3. Sketch the graph 

4. Values that satisfy the inequality $x \leq -8$, $x \geq 3$

Turning point and roots of a quadratic equation



Straight line graphs

$y = mx + c$

$m = gradient$

$c = y - \ intercept$

positive gradient

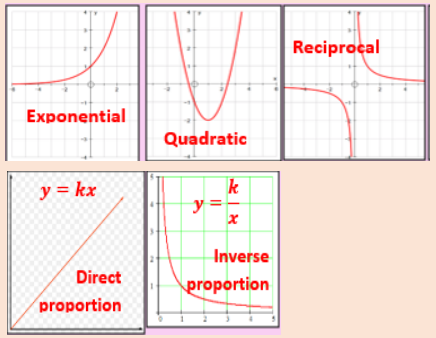
negative gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{change\ in\ y}{change\ in\ x}$$

Parallel lines – have equal gradients

Perpendicular lines – If L_1 and L_2 are perpendicular then $m_2 = -\frac{1}{m_1}$

Graphs that need to be recognised:



Equation of a circle centre (0, 0)

$x^2 + y^2 = r^2$

Functions

$f(4)$: *Substitute 4 into the function*

$f(g(x))$: *Substitute* $g(x)$ *into* $f(x)$ i.e. replace all values of x in $f(x)$ with the entire function $g(x)$

e.g. $f(x) = 2x + 3$, $g(x) = x - 3$, $fg(x) = 2(x-3) + 3$

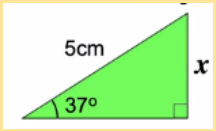
Geometry and measure – Higher Part 1

Trigonometry

$$S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$$

Example – finding a side:

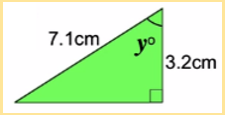
$\sin 37 = \frac{x}{5}$



x = 5 x sin 37^0

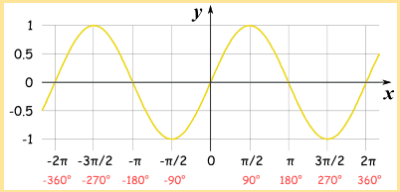
Example – finding a side:

$\tan y = \frac{3.2}{7.1}$

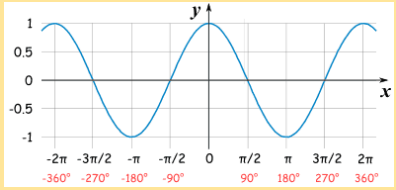


$y = tan^{-1}\left(\frac{3.2}{7.1}\right)$.

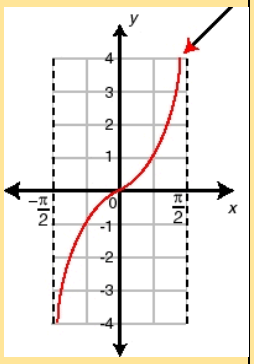
Sine Curve



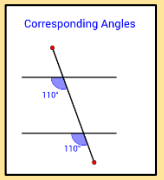
Cosine Curve

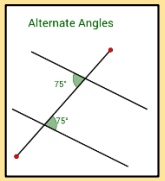


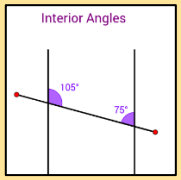
Tangent Curve



Angles in parallel lines







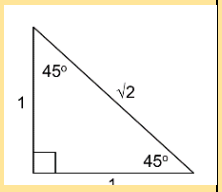
Corresponding angles are equal

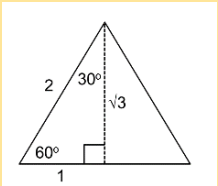
Alternate angles are equal

Co-interior angles are equal

Exact Trig values

Angle (θ)	sin(θ)	cos(θ)	tan(θ)
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined





Simple vector notation

$$\begin{pmatrix} a \\ b \end{pmatrix}$$

a: movement along the x-axis (left or right)

b: movement along the y-axis (up or down)

$-a$: movement left $\quad\quad$ $-b$: movement down

Operations with vectors

$\begin{pmatrix} 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 7 \\ -3 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$

If $b = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, then $3b = \begin{pmatrix} 12 \\ -6 \end{pmatrix}$

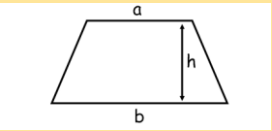
Volume & surface area

Learn the cylinder

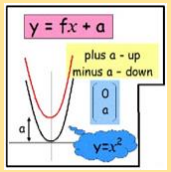
$$V = \pi r^2 h$$
$$SA = 2\pi r^2 + \pi dl$$

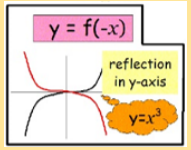
Area of a trapezium

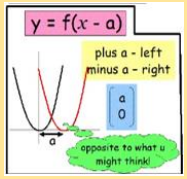
$$A = \frac{1}{2}(a+b)h$$

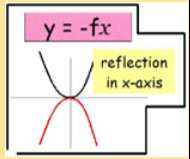


Transformation of a graph





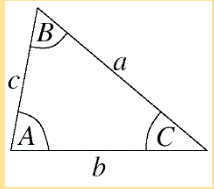




Sine rule

angles: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

sides: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



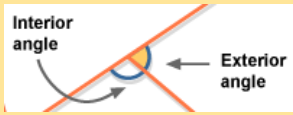
Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Area of a triangle

$$\frac{1}{2}ab\sin C$$

Angles in regular polygons



$n = number\ of\ sides$

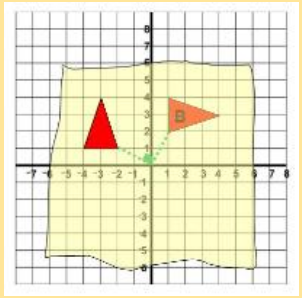
Interior angle + exterior angle = 180^0

Exterior angle = $\frac{360}{n}$

$$n = \frac{360}{Exterior\ angle}$$

Geometry and measure – Higher Part 2

Transformations – rotation – describing:



Always use tracing paper.
Describe:

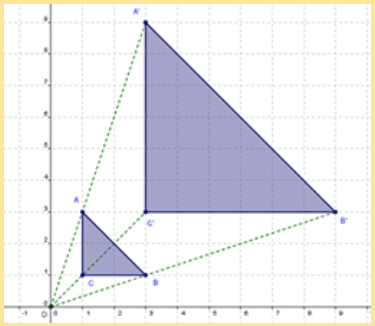
1. It's a rotation
2. Size of rotation in degrees
3. Orientations: clockwise or anticlockwise
4. Centre of rotation given as a coordinate (x,y)

Transformation – translation

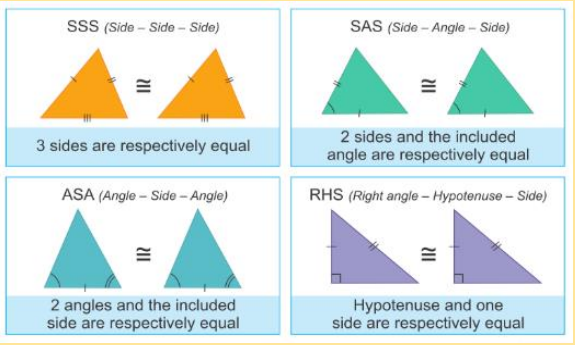
Vector $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$ 6 right, 4 down

Transformations – enlargement - describing:

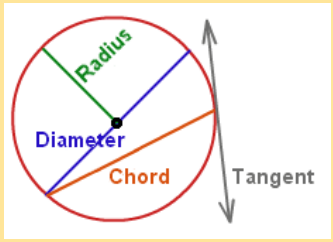
1. It's an enlargement
2. The scale factor (if the image is smaller than the object the scale factor is fractional e.g. ½)
3. The centre of enlargement given as a coordinate

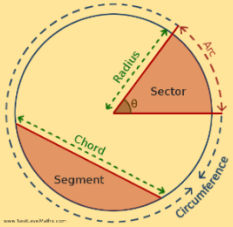


Congruent triangles



Circles





$Area = \pi r^2$

$Circumference = \pi d$

Sector $Area = \frac{\theta}{360}\pi r^2$

Arc length = $\frac{\theta}{360}\pi d$

Similar shapes

Same shape, different sides
The ratio of the lengths of corresponding sides are equal

Length scale factor = x

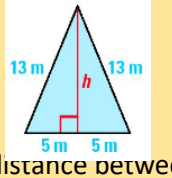
Area scale factor = x^2

Volume scale factor = x^3

Pythagoras' Theorem

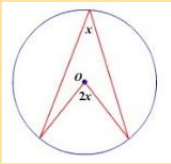
$$a^2 + b^2 = c^2$$

Only applies to right angled triangles.
Can be used to find the height of an isosceles triangle

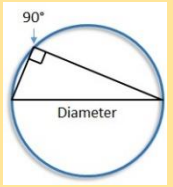


Can be used to find the length distance between two coordinates

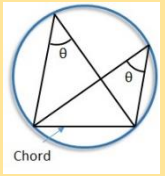
Circle Theorems



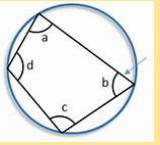
Angle at the centre is twice the angle at the circumference



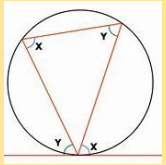
Angles in a semicircle are 90°.



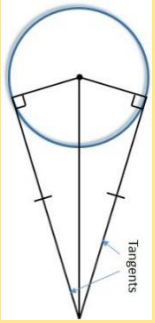
Angles in the same segment are equal.



Opposite angles of a cyclic quadrilateral add up to 180).



Alternate segment theorem.



Tangents from an external point are equal in length.

The tangent to a circle is perpendicular (90°) to the radius

Number Ratio and Proportion – Higher Part 1

Estimate
Round each value to one significant figure

Standard form
$a \times 10^n$, where $1 \leq a < 10$

Reciprocal
Reciprocal of 7 is $\frac{1}{7}$, reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ etc

Sequences
Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21
Geometric Sequence: each term is multiplied but he same constant to get the next number.
E.g. 3, 12, 48, 191, …. (x by 4 each time)

Simplifying Surds
Find a factor that is a square number

$$\sqrt{96} = \sqrt{16 \times 6} = 4\sqrt{6}$$

Manipulating surds

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Rationalising Surds
Rationalise by removing any surds from the denominator
E.G with surd.

$$\frac{2\sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{3} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{2\sqrt{3 \times 5}}{\sqrt{5 \times 5}} = \frac{2\sqrt{15}}{\sqrt{25}} = \frac{2\sqrt{15}}{5}$$

E.G with surd expressions multiply by top and bottom by the denominator with the opposite sign.

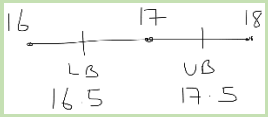
$$\frac{5}{3+\sqrt{2}} = \frac{5 \times (3-\sqrt{2})}{(3+\sqrt{2}) \times (3-\sqrt{2})} = \frac{5(3-\sqrt{2})}{9-\sqrt{4}}$$

$$= \frac{5(3-\sqrt{2})}{7}$$

Recurring Decimals
Form two equations where the digits following the decimal point are the same, and therefore can be cancelled

Upper and lower bounds
Look at the value above and below for the same place value. LB and UB will be half way between these points

e.g. 17 rounded to the nearest integer



e.g. 24.6 roudned to one decimal place.
LB = 24.55, UB = 24.65

Fractions

Add and Subtract – ensure the fractions have the same denominator before adding numerators

$$\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$$

Multiply – multiply numerators and denominators

$$\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

Divide – take reciprocal of the second fraction and then multiply the new numerators and denominators

$$\frac{4}{5} \div \frac{1}{3} = \frac{4}{5} \times \frac{3}{1} = \frac{12}{5} = 2\frac{2}{5}$$

Percentages

Finding percentages of an amount

1%	÷100
5%	÷20
20%	÷5
25%	÷4
50%	÷2

Multipliers:
To find the multiplier for a percentage, divide by 100

Use multipliers on a calculator paper
e.g. 35% of 370 = 0.35 x 370

Increasing and decreasing a given amount
Calculator:

$$Orginal\ Amount\ x\ mutiplier = new\ amount$$

Non-calculator: find the increase or decrease and add to the original amount

Finding percentage increase or decrease (profit/loss)

$$\frac{value\ of\ increase/decrease}{Original} \times 100$$

Writing an amount as a percentage of the original

$$\frac{Amount}{Original} \times 100$$

Reverse Percentage – finding the original amount

$$Orginal\ Amount = \frac{New\ Amount}{multiplier}$$

Number Ratio and Proportion – Higher Part 2

Growth & Decay / Compound interest

$$original\ amount \times multiplier^{time}$$

Where the multiplier is the percentage, increase or decrease from 100%, converted to a decimal.
e.g.
30% decrease is 70% = 0.7
30% increase is 130% = 1.3

Dividing by decimals:

1. Write the calculation as a fraction
2. Form an equivalent fraction to makes integers (multiply by powers of 10)
3. Use short division (bus stop) to calculate

e.g. $460 \div 0.4 = \frac{460}{0.4} = \frac{4600}{4} = 1150$

Conversions

10 millimetres = 1 centimetre	15 minutes = 0.25 hours
100 centimetres = 1 metre	30 minutes = 0.5 hours
1000 metres = 1 kilometre	45 minutes = 0.75 hours
$1000cm^3$ = 1 litre	1000g = 1 kilogram
1000ml = 1 litre	1000kg = 1 tonne

Compound Units (rearrange as necessary)

$$Speed = \frac{Distance}{Time}$$

$$Area = \frac{Force}{Pressure}$$

$$Density = \frac{Mass}{Volume}$$

Error Intervals

least possible value $\leq x <$ greatest possible value

e.g. A fence is 30 m long to the nearest 10 m.

$25\text{ m} \leq l < 35\text{ m}$

Truncation

Truncation is a method of approximating a decimal number by dropping all decimal places past a certain point **without** rounding.

e.g. Truncate 3.14159265 to 4 decimal places.
= 3.1415

Negative numbers

Adding and subtracting: (vertical number lines help)
-3 – 5 = - 8
-3 + 5 = 2
-3 - - 5 = -3 + 5 = 2
-3 - + 5 = -3 – 5 = -8
-3 + - 5 = -3 – 5 = -8

Multiplying and dividing:
Different signs – answer will be negative
+ x - = -, - x + = -
Same signs – answer will be positive
- x - = +

Product rule

If there are ***m*** ways to do one thing and ***n*** ways to do another, then there are ***m*** x ***n*** ways to do *both*

Order of operations

Bracket
Indices
Division and **M**ultiplication
Addition and **S**ubtraction

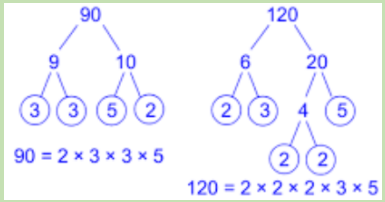
Rounding to significant figures

Start from the first **non-zero** number and round as normal, but ensure the place value is correct
e.g. 345,635 to 2SF = 350,000
0.0060821 to 3SF = 0.0608

Index Laws

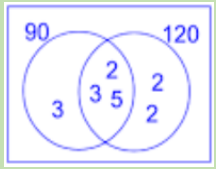
$$a^n \times a^m = a^{n+m}$$
$$a^n \div a^m = a^{n-m}$$
$$(a^n)^m = a^{nm}$$
$$a^0 = 1$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Prime Factorisation



HCF and LCM of 90 and 120 (Factor Tree & Venn Diagram)

HCF is the product of common factors
LCM is the product of common factors and remaining factors.



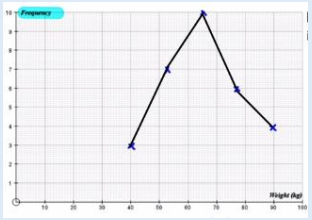
HCF: 2x3x5
LCM: 2^3x3^2x5

Probability and Statistics - Higher

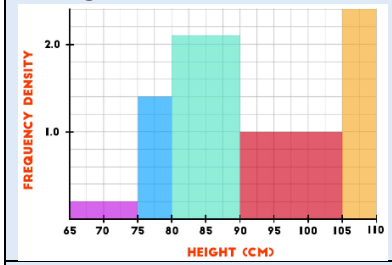
Frequency Polygons

1. Plot frequency at the mid-point
2. Join with straight lines

Weight w (kg)	Frequency
$30 \leq w < 50$	3
$50 \leq w < 55$	7
$55 \leq w < 75$	10
$75 \leq w < 80$	6
$80 \leq w < 100$	4



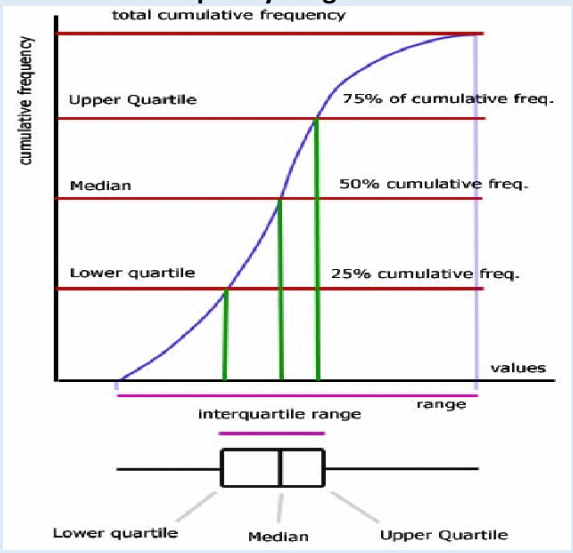
Histograms



FD = Frequency density

$$FD = \frac{Frequency}{Class\ Width}$$

Cumulative Frequency Diagrams and Box Plots



Averages from a frequency table

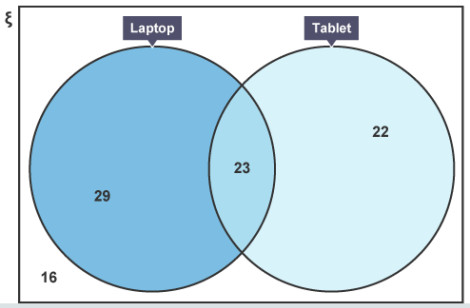
Mean: $\frac{\sum fw}{\sum f}$; where, w is the midpoint of the group.

Median group: find which group the $\frac{n+1}{2}th$, value lies.
Where, n is the total frequency.

E.G. in this table 51.5th value which lies in group $8 < w \leq 12$ (using the cumulative frequency

Weight of box (w kg)	Frequency
$0 < w \leq 4$	11
$4 < w \leq 8$	16
$8 < w \leq 12$	29
$12 < w \leq 16$	26
$16 < w \leq 20$	20

Venn Diagrams



Information given:
90 pupils were surveyed
52 said they owned a laptop.
45 said they owned a tablet.
23 said they owned both.

Notation

A – all elements **in A**
A' – all elements **not in A**
B – all elements **in B**
B' – all elements **not in B**
A U B – all the elements **in A or B or both**
A ∩ B – all the elements **in both A and B**

Expected outcomes

Relative frequency: $frequency \div total\ trials$

Expected outcome = probability x number of trials

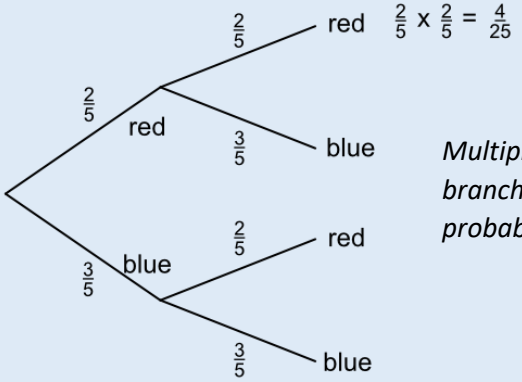
E.g. A biased spinner is spun 800 times. The probabilities is lands on each colour is below. The probability of it landing on red is the same as the probability of it landing on green. How many times would you expect yellow to come up.

Result	Red	Green	Brown	Yellow
Probability		0.48	0.2	

P(Y) = (1 – 0.48 – 0.2) ÷ 2 = 0.32 ÷ 2 = 0.16

Expected yellow = 0.16 x 800 = 128

Tree diagrams



Multiply along the branches to find each probability.

1. Probability that a red counter is picked both times $P(RR) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

2. Probability that the counters are different colours = $P(RB) + P(BR) = \frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} = \frac{12}{25}$